THEORY OF COMPUTATION :-

SLIP - TEST

1) Diff. L^* and L^+

$L^* \Rightarrow L^*$ refers to Kleen Closure :- Here we will use the ϵ for connecting first to final state, and final before state to initial next state.

$L^+ \Rightarrow L^+$ refers to 'positive closure'. The difference here is that we will not use 'ϵ' for connecting the initial to final state.

2) All string that contains exactly 4 zeros.

Sol: Step 1:

$M = \{Q, \Sigma, \delta, q_0, F\}$

$w = 0000$

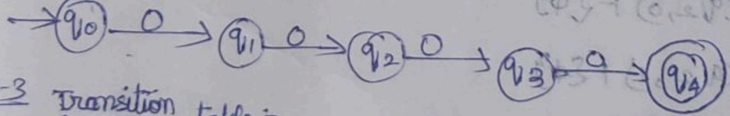
$Q = \{q_0, q_1, q_2, q_3, q_4\}$

F = Final state $= \{q_4\}$

$q_0 \Rightarrow$ Initial state $= \{q_0\}$

Step 2 :-

Diagram :-



Step-3 Transition table :-

	0	1
$\rightarrow q_0$	$\{q_1\}$	$\{\phi\}$
q_1	$\{q_2\}$	$\{\phi\}$
q_2	$\{q_3\}$	$\{\phi\}$
q_3	$\{q_4\}$	$\{\phi\}$
$* q_4$	$\{\phi\}$	$\{\phi\}$

v) Tak
vi) 1

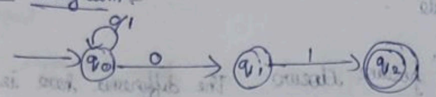
Step 4 :- String verification :

$\delta(q_0, 0000) \vdash \delta(q_1, 000) \vdash \delta(q_2, 00) \vdash \delta(q_3, 0)$
$\vdash \delta(q_4)$

PART - B

vii)
viii)
ix)

2) Given diagram :-



Steps :-

Step 1 :- First construct the transition function for the following NFA wo/o ε diagram

Step 2 :- Then convert the transition function into DFA diagram :-

Step 1 :- Transition function :-

$\delta(q_0, 0) \vdash \{q_0, q_1\} \rightarrow N_1$

$\delta(q_0, 1) \vdash \{q_0\}$

$\delta(q_1, 0) \vdash \{\phi\}$

$\delta(q_1, 1) \vdash \{q_2\}$

$\delta(q_2, 0) \vdash \{\phi\}$

$\delta(q_2, 1) \vdash \{\phi\}$

$\delta(N_1, 0) \vdash (\delta(q_0, q_1) \cdot 0))$

$\vdash (\delta(q_0, 0) \cup \delta(q_1, 0))$

$\vdash (q_0, q_1) \cup (\phi))$

$\boxed{\delta(N_1, 0) \vdash \{q_0, q_1\}} \rightarrow N_1$

$\delta(N_1, 1) \vdash \delta((q_0, q_1) \cdot 1))$

$\vdash \delta(q_0, 1) \cup \delta(q_1, 1))$

$\vdash (q_0) \cup (q_2)$

$\vdash \{q_0, q_2\} \rightarrow N_2$

$\delta(N_2, 0) \vdash \delta((q_0, q_2) \cdot 0))$

$\vdash \delta(q_0, 0) \cup \delta(q_2, 0)$

$\vdash \{q_0, q_1\} \cup \{\phi\}$

$\boxed{\delta(N_2, 0) \vdash \{q_0, q_1\}} \rightarrow N_1$

$\delta(N_2, 1) \vdash \delta((q_0, q_2) \cdot 1))$

$\vdash \delta(q_0, 1) \cup \delta(q_2, 1))$

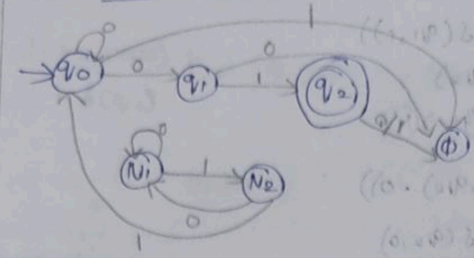
$\vdash (q_0) \cup (\phi)$

$\boxed{\delta(N_2, 1) \vdash \{q_0\}}$

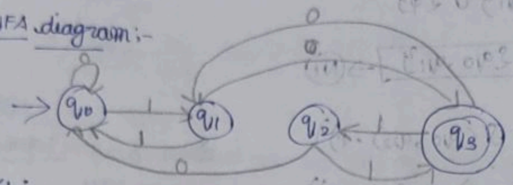
Transition table:

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{\phi\}$
q_1	$\{\phi\}$	$\{q_2\}$
$* q_2$	$\{\phi\}$	$\{\phi\}$
N_1	$\{q_0, q_1\}$ N_1	N_2
N_2	N_1	q_0

DFA diagram :-



4] NFA diagram :-



String verification :-

$\cancel{\delta(101101) \vdash \delta}$

$\delta(q_0, 101101) \vdash \delta(q_1, 01101) \vdash \delta(q_3, 1101) \vdash \delta(q_2, 101)$
$\vdash \delta(q_3, 01) \vdash (q_1, 1) \vdash (q_0)$

$\delta(q_0, 11111) \vdash \delta(q_1, 1111) \vdash \delta(q_0, 111) \vdash \delta(q_1, 11) \vdash \delta(q_0, 1)$
$\vdash \delta(q_1)$

Since the strings 101101 and 11111 are ~~acce~~ not accepted by M.

Extended Transition function for NFA:

NFA definition

* NFA stands for non-deterministic finite automata.
* The finite automata are called NFA when there exist many paths for specific input from the current state to the next state

NFA functions:

NFA consists of five tuple function, $M = \{Q, \Sigma, \delta, q_0, F\}$

$Q \Rightarrow$ Finite set of state which is not empty

$\Sigma \Rightarrow$ is a input alphabet, indicates input set

$\delta \Rightarrow$ transition function or matrix function.

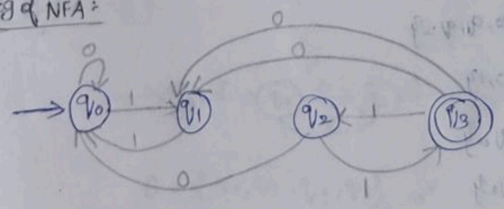
For NFA $\rightarrow$ $\delta = Q \times 2^Q$ [which is extended transition function]

$q_0 \Rightarrow$ initial state and q_0 is in Q

ie) $q_0 \in Q$.

$F \Rightarrow$ It is the set of final state.

Eg of NFA:



$q_0 \Rightarrow$ Initial state

$F \Rightarrow q_3 \Rightarrow$ Final state

$\Sigma = \{0, 1\}$

$Q = \{q_0, q_1, q_2, q_3\}$

DFA definition:

* DFA stands for Deterministic finite automata which accepts or reject strings of characters by parsing them through a sequence that is uniquely determined by each string.

DFA function:

DFA consists of a collection of 5 tuples $M = \{Q, \Sigma, \delta, q_0, F\}$

where $Q \Rightarrow$ finite set of state which is not empty.

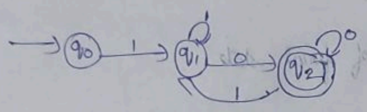
$\Sigma \Rightarrow$ is a input alphabet which indicates input set.

$\delta \Rightarrow$ transition function or matrix function.
Here transition function is $\delta \Rightarrow Q \times \Sigma$

$q_0 \Rightarrow$ initial state and q_0 is in Q

$F \Rightarrow$ It is the set of final state

Eg of DFA:



$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1\}$

$I.S = \{q_0\}$

$F = \{q_2\}$

NFA-ε definition :

$\Rightarrow$ ε denotes a regular expression which denotes the set $\{\varepsilon\}$ and it is a null string.

$\Rightarrow$ The Non-deterministic finite automata can be expressed with the help of ε.

NFA-ε functions :

NFA-ε consists of a collection of 5 tuples $M = \{Q, \Sigma, \delta, q_0, F\}$

where $Q \Rightarrow$ finite set of state which is not empty.

$\Sigma \Rightarrow$ is a input alphabet which indicates input set.

$\delta \Rightarrow$ transition function or matrix function.

Here Extended transition function is

$$\delta \Rightarrow Q \times 2^{Q}$$

$q_0 \Rightarrow$ Initial state and q_0 is in Q.

$F \Rightarrow$ It is the set of Final state.

Eg. of NFA-ε :-

$$\begin{tikzcd} \arrow[r] & q_2 \arrow[r, "\varepsilon"] & q_0 \arrow[r, "0"] & q_1 \arrow[r, "\varepsilon"] \arrow[l, bend left, "\varepsilon"] & q_3 \end{tikzcd}$$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$

$I.S \Rightarrow \{q_2\}$

$F = \{q_3\}$

Set - B (QP)

1) Principle of Mathematical Induction :-

Mathematical Induction is a technique of proving a statement, theorem or formula which is taught to be true for each and every natural number n.

Here Mathematical induction proof consists of 3 steps,

i) Basis of Induction

ii) Inductive Hypothesis

iii) Inductive step

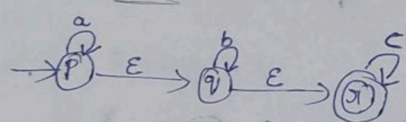
2) Diff Kleen closure Positive closure :

Kleen closure - The r^* is known as kleen closure which indicates occurrence of r infinite no. of times which has 'ε' included in it.

Positive closure - It indicates a set of strings without a null string (ε)

PART - B

3) ε-NFA diagram :



ε - closure of

ε-closure (P) - {p, q, r}

ε - closure (q) - {q, r}

ε - closure (r) - {r}.

Conversion of NFA with ϵ to NFA w/o ϵ:

$\delta(p, a) \vdash \epsilon\text{.closure}(\delta(p, a))$

$\vdash \epsilon\text{.closure}(\delta(\epsilon\text{.closure}(p), a))$

$\vdash \epsilon\text{.closure}(\delta\{p, q, r\}, a))$

$\vdash \epsilon\text{.closure}(\delta(p, a) + \delta(q, a) + \delta(r, a)\}$

$\vdash \epsilon\text{.closure}(\{p\} \cup (\phi) \cup (\phi)$

$\boxed{\delta(p, a) \vdash \{p, q, r\}}$

$\delta(p, b) \vdash \epsilon\text{.closure}(\delta(p, b))$

$\vdash \epsilon\text{.closure}(\delta(\epsilon\text{.closure}(p), b))$

$\vdash \epsilon\text{.closure}(\delta(\{p, q, r), b))$

$\vdash \epsilon\text{.closure}(\delta(p, b) \cup \delta(q, b) \cup \delta(r, b))$

$\vdash \epsilon\text{.closure}((\phi) \cup (q) \cup (\phi)\}$

$\boxed{\delta(p, b) \vdash \{q, r\}}$

$\delta(p, c) \vdash \epsilon\text{.closure}(\delta(p, c))$

$\vdash \epsilon\text{.closure}(\delta(p, c) \cup \delta(q, c) \cup \delta(r, c))$

$\vdash \epsilon\text{ closure}(\phi) \cup (\phi) \cup (r))$

$\boxed{\vdash(p, c) \vdash \{r\}}$

$\delta(q, a) \vdash \epsilon\text{ closure}(\delta(q, a))$

$\vdash \epsilon\text{.closure}(\delta(\epsilon\text{.closure}(q), a))$

$\vdash \epsilon\text{.closure}(\delta(q, r), a))$

$\vdash \epsilon\text{.closure}(\delta(q, a) \cup \delta(r, a))$

$\vdash \epsilon\text{.closure}(\phi) \cup (\phi)$

$\boxed{\delta(q, a) \vdash \{\phi\}}$

$\delta(q,b) \vdash \varepsilon\text{ closure } (\delta(q,b))$

$\vdash \varepsilon\text{ closure } (\delta(q,b) \cup \delta(r,b))$

$\vdash \varepsilon\text{ closure } ((q) \cup (\phi))$

$\boxed{\delta(q,b) \vdash \{q, r\}}$

$\delta(q,c) \vdash \varepsilon\text{-closure } (\delta(q,c))$

$\vdash \varepsilon\text{-closure } (\delta(q,c) \cup \delta(r,c))$

$\vdash \varepsilon\text{ closure } ((\phi) \cup (r))$

$\boxed{\delta(q,c) \vdash \{r\}}$

$\delta(r,a) \vdash \varepsilon\text{ closure } (\delta(r,a))$

$\vdash \varepsilon\text{-closure } (\delta(\varepsilon\text{-closure } (r) \cdot a))$

$\vdash \varepsilon\text{-closure } (\delta(r) \cdot a))$

$\vdash \varepsilon\text{-closure } (\delta(r,a))$

$\vdash \varepsilon\text{-closure } (\phi)$

$\boxed{\delta(r,a) \vdash \{\phi\}}$

$\delta(r,b) \vdash \varepsilon\text{-closure } (\delta(r,b))$

$\vdash \varepsilon\text{-closure } (\delta(r,b))$

$\boxed{\delta(r,b) \vdash \{\phi\}}$

$\delta(r,c) \vdash \varepsilon\text{ closure } (\delta(r,c))$

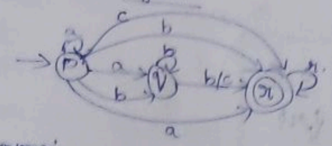
$\vdash \varepsilon\text{ closure } (\delta(r,c))$

$\boxed{\delta(r,c) \vdash \{r\}}$

Transition table

	a	b	c
$\rightarrow p$	$\{p, q, r\}$	$\{q, r\}$	$\{r\}$
q	$\{\phi\}$	$\{q, r\}$	$\{r\}$
$*r$	$\{\phi\}$	$\{\phi\}$	$\{r\}$

NFA w/o ε diagram:



Conversion to DFA diagram:

i) Transition function:

$\delta(p,a) \vdash \{p,q,r\} \to N_1$

$\delta(p,b) \vdash \{q,r\} \to N_2$

$\delta(p,c) \vdash \{r\}$

$\delta(q,a) \vdash \{\phi\}$

$\delta(q,b) \vdash \{q,r\} \to N_2$

$\delta(q,c) \vdash \{r\}$

$\delta(r,a) \vdash \{\phi\}$

$\delta(r,b) \vdash \{\phi\}$

$\delta(r,c) \vdash \{r\}$

~~$\delta(N_1,a) \vdash$ ε closure ($\delta(N_1,a)$)~~

~~$\vdash$ ε closure (δ(ε closure~~

$\delta(N_1,a) \vdash \delta((p,q,r),a))$

$\vdash \delta(p,a) \cup \delta(q,a) \cup \delta(r,a)$

$\vdash \{p,q,r\} \cup \{\phi\} \cup \{\phi\}$

$\boxed{\delta(N_1,a) \vdash \{p,q,r\}} \to N_1$

$\delta(N_1,b) \vdash \delta((p,q,r),b))$

$\vdash \delta(p,b) \cup \delta(q,b) \cup \delta(r,b)$

$\vdash \{q,r\} \cup \{q,r\} \cup \{\phi\}$

$\boxed{\delta(N_1,b) \vdash \{q,r\}} \to N_2$

$\delta(N_1,c) \vdash \delta(\{p,q,r\}.c))$

$\vdash \delta(p,c) \cup \delta(q,c) \cup \delta(r,c))$

$\vdash \{r\} \cup \{r\} \cup \{r\}$

$\boxed{\delta(N_1,c) \vdash \{r\}}$

$\delta(N_2,a) \vdash \delta((q,r).a))$

$\vdash \delta(q,a) \cup \delta(r,a))$

$\vdash \{\phi\} \cup \{\phi\}$

$\boxed{\delta(N_2,a) \vdash \{\phi\}}$

$\delta(N_2,b) \vdash \delta((q,r).b))$

$\vdash \delta(q,b) \cup \delta(r,b)$

$\vdash \{q,r\} \cup \{\phi\}$

$\boxed{\delta(N_2,b) \vdash \{q,r\}} \to N_2$

$\delta(N_2,c) \vdash \delta((q,r),c))$

$\vdash \delta(q,c) \cup \delta(r,c)$

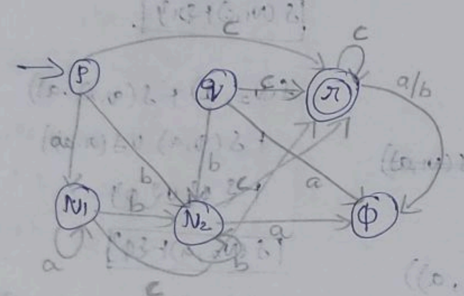
$\vdash \{r\} \cup \{r\}$

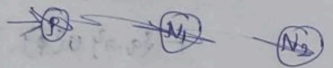
$\boxed{\delta(N_2,c) \vdash \{r\}}$

Transition table :-

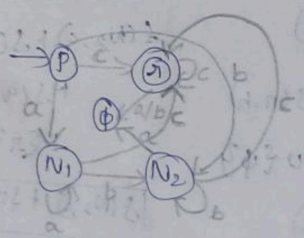
	a	b	c
→p	N_1	N_2	{r}
q	{φ}	N_2	{r}
*r	{φ}	{φ}	{r}
N_1	{N_1}	{N_2}	{r}
N_2	{φ}	{N_2}	{r}

DFA diagram :





Minimalized DFA diagram :



1) Given $(0+1)^* (00+11)(0+1)^*$

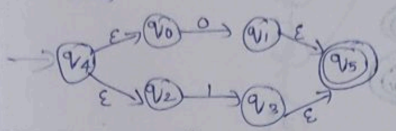
Sol:

$RE_1 = RE_{11} + RE_{12}$

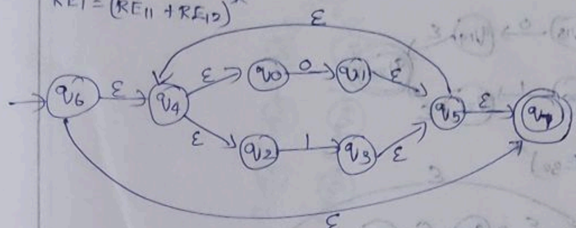
$RE_{11} = 0 \Rightarrow \rightarrow q_0 \xrightarrow{0} q_1$

$RE_{12} \Rightarrow 1 \Rightarrow \rightarrow q_2 \xrightarrow{1} q_3$

$RE_1 \Rightarrow \cancel{RE}$



$RE_1 = (RE_{11} + RE_{12})^*$



$RE_2 \Rightarrow RE_{21} + RE_{22}$

$RE_{21} \Rightarrow 0 \cdot 0$

$0 \Rightarrow \rightarrow q_8 \xrightarrow{0} q_9$

$0 \Rightarrow \rightarrow q_{10} \xrightarrow{0} q_{11}$

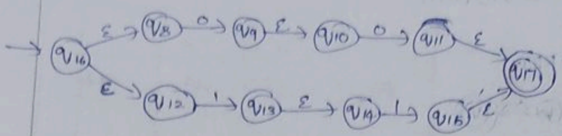
$RE_{21} \Rightarrow \rightarrow q_8 \xrightarrow{0} q_9 \xrightarrow{\varepsilon} q_{10} \xrightarrow{0} q_{11}$

$RE_{22} \Rightarrow 1 \Rightarrow \rightarrow q_{12} \xrightarrow{1} q_{13}$

$1 \Rightarrow \rightarrow q_{14} \xrightarrow{1} q_{15}$

$RE_{22} \Rightarrow 1 \cdot 1 \Rightarrow \rightarrow q_{12} \xrightarrow{1} q_{13} \xrightarrow{\varepsilon} q_{14} \xrightarrow{1} q_{15}$

$RE_2 \Rightarrow RE_{21} + RE_{22}$

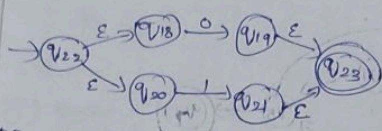


$RE_3 \Rightarrow RE_{31} + RE_{32}$

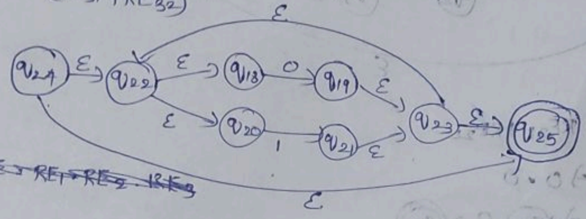
$RE_{31} \Rightarrow 0 \quad \rightarrow q_{18} \xrightarrow{0} q_{19}$

$RE_{32} \Rightarrow 1 \quad \rightarrow q_{20} \xrightarrow{1} q_{21}$

$RE_3 \rightarrow$



$RE_3 = (RE_{31} + RE_{32})^*$



~~RE = RE1 + RE2 . RE3~~

$RE = RE_1 . RE_2 . RE_3$